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The Impact-Induced Triggering of Hot Spots in Energetic/Explosive Materials Part 2: Adiabatic Temperature Distribution Near a Spherical Hole

by Michael Grinfeld and Todd Bjerke

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14. ABSTRACT Currently, there is a considerable gap in the level of theoretical understanding of the mechanisms triggering energy release between liquid and solid energetic/explosive materials. Although adiabatic impact-induced triggering in solid energetic /explosive materials is still treated similarly to liquid, there are qualitative differences in the impact-induced triggering of liquids and solids. In this report, we discuss our recent results related to the impact-induced triggering in solid energetic materials.					
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Contents

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1. General Introduction

Solid vs. Liquid Triggering in Energetic/Explosive Substances

Triggering (initiating) is one of the key processes in combustion, explosion, and detonation of energetic/explosive materials. By controlling the triggering mechanisms, these strong processes can be made more probable or suppressed if necessary. In order to get such control, one has to clearly understand the physical nature of the triggering mechanisms in different substances. Depending on the substance, the triggering mechanisms can be quite different.

The processes of triggering hot spots by adiabatic loading of liquid energetic/explosive materials have been explored quite well (e.g., the classical monographs in references *1–4*). It was established—both theoretically and experimentally—that the key role in these processes is played by various gaseous bubbles (figure 1). Because the bubbles are elastically much softer than the surrounding energetic/explosive liquids, they reach much higher temperatures than the liquids under the same impact pressure. Thus, they are the hot spots for initiating corresponding exothermal chemical reactions.

In several respects, the impact-induced adiabatic triggering in energetic/explosive solids differs from the corresponding liquid materials. Three essentially new circumstances appear when dealing with triggering hot spots in solid energetic/explosive substances:

1. At equilibrium, liquid substance is able to sustain hydrostatic loading only, whereas all spectrum of nonhydrostatic loading becomes possible in solids.
2. Also at equilibrium, the gaseous bubbles in liquids are always spherical due to the surface tension. In solids, the dissolved gases or vapors accumulate within the pre-existed defects of various shapes: voids, cracks, intergrain spaces, etc.

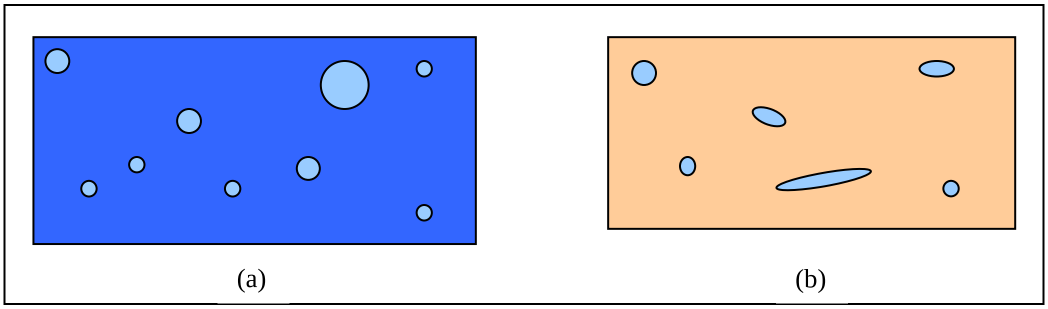

Figure 1. Bubbles with gas within liquid (a) and solid (b) energetic/explosive substances.

3. The energetic/explosive liquids completely transmit the externally applied pressure to the gaseous bubbles dissolved within them. When dealing with gaseous bubbles in energetic/explosive solids, the solid matrix withstands itself to the externally applied transmitted loads.

In our studies, we sequentially analyze each of these factors and their implications. In particular, we dwell on a detailed exploration of the effect of adiabatic loading of the solid media with a spherical hole possessing another substance within itself. This study gives theoretical and engineering insight into the mechanisms of generating hot spots in solid energetic/explosive materials.

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2. The Exact Nonlinear Systems of Adiabatic Loading and Its Linearized Version

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2.1 Geometrical Settings

The geometry of the system to be analyzed is shown in figure 2. The entire space is referred to as the Cartesian coordinate x^i, with the basis covariant basis $\vec{x}_i$ * and contravariant basis $\vec{x}^i$.

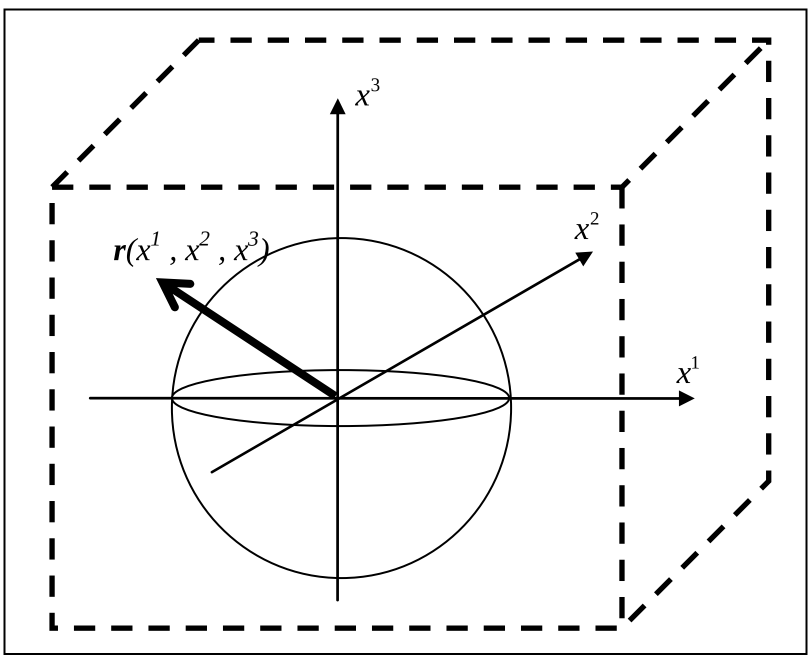

Figure 2. The geometry of the system.

*Although the systematic use of "tensorial" grammar and language are not particularly important when using the Cartesian coordinates, the formulas look much more elegant and well structured when this grammar is respected. This grammar allows the Cartesian coordinate system to be replaced with others. The reader can ignore all the tensorial adjectives without losing essential physics. Note that in the Cartesian coordinates systems, the covariant and contravariant components of all tensors are numerically equal.

The Lagrangian (material) coordinates of the body's particles in the unstressed reference configuration coincide with the corresponding coordinates of the space. We use the notation of $\vec{r}(x^i) \equiv x^i \vec{x}_i$ for the radius vector, with the pole at the origin of the coordinate system. Let $\vec{n} = n_i \vec{x}^i = n^i \vec{x}_i$ be the components of the unit normal to the hole's boundary in the reference configuration. For the spherical hole centered at the origin, the components of the unit are given by the formula $n^i = x^i \left| x^i \right|$ (no summation!). Let $\vec{u}(x) = u^i(x)\vec{x}_i = u_i(x)\vec{x}^i$ be the displacements of the material point with the Lagrangian coordinates x^i.

The analysis in this report deals with thermoelastic stresses within an infinite isotropic matrix possessing a hole and exposed to the specified displacement gradients κ_{ij} at infinity. In other words, the displacements at infinity are given by the following formula:

$$u_i \to \kappa_{ij} x^j \text{ at } |x^m| \to \infty \ . \tag{1}$$

2.2 Some Necessary Elements of Thermodynamics

In our notation of thermomechanical notions, we follow the monographs in references *5–8*. Local thermodynamic state of any elastic solid can be fixed by the pairs $(u_{i,j}, \eta)$ or $(u_{i,j}, \theta)$, where $u_{i,j}$ is the displacement gradient, η is the specific* entropy density, and θ is the absolute temperature. This statement means that all other thermodynamic functions are uniquely determined by any one of these pairs. It is particularly true regarding the internal energy density e and the free energy density ψ. If the energy density e is given as a function of $(u_{i,j}, \eta)$ or if the energy density ψ is known as function of $(u_{i,j}, \theta)$, i.e.,

$$e \equiv e(u_{i,j}, \eta),\ \psi = \psi(u_{i,j}, \theta)\ , \tag{2}$$

then any other thermodynamic function can be calculated only by differentiation and solving some algebraic equations.

The Piola-Kirchhoff stress tensor p^{ji} can be treated as a function of $(u_{i,j}, \eta)$ – $p^{ji}(u_{k,l}, \eta)$ or $(u_{k,l}, \theta)$ – $p^{ji}(u_{k,l}, \theta)$. Of course, it is silently assumed that the two functions are completely different. Sometimes, in order to avoid possible confusion, we use the notation ${}_{\eta}p^{ji}$ for the first function and ${}_{\theta}p^{ji}$ for the second function.

The first and second laws of thermodynamics imply the following identities:

$${}_{\eta}p^{ij}(u_{k,l}, \eta) \equiv \frac{\partial e(u_{k,l}, \eta)}{\partial u_{i,j}},\ \theta(u_{k,l}, \eta) \equiv -\frac{\partial e(u_{k,l}, \eta)}{\partial \eta}\ , \tag{3}$$

and

* In the following, the adjective "specific" means "per unit volume in the reference configuration."

$$ {}_{\theta}p^{ij}(u_{k,l},\theta) \equiv \frac{\partial \psi(u_{k,l},\theta)}{\partial u_{i,j}},\ \eta(u_{k,l},\theta) \equiv \frac{\partial \psi(u_{k,l},\theta)}{\partial \theta}\ . \tag{4} $$

The displacement gradients and the absolute temperature are usually the most convenient quantities for direct experimental measurements. That is why the function $\psi(u_{k,l},\theta)$ plays such an important role in experimental and theoretical thermodynamics. Its first derivatives can then be interpreted with the help of the thermodynamic identities in equation 4. It is much harder to measure the Piola-Kirchhoff stress tensor and the entropy density.

The second derivatives of the free energy define the so-called instantaneous isothermal elastic modules ${}_{\theta}c^{ijkl}$, the instantaneous tensor of thermal expansions β^{ij}, and the instantaneous heat capacity c_u at fixed displacements:

$$ {}_{\theta}c^{ijkl} \equiv \frac{\partial^2 \psi(u_{m,n},\theta)}{\partial u_{i,j}\partial_{k,l}},\ \beta^{ij} \equiv \frac{\partial^2 \psi(u_{m,n},\theta)}{\partial u_{i,j}\theta} = \frac{\partial p^{ji}(u_{m,n},\theta)}{\partial \theta}, $$
$$ c_u \equiv -\theta \frac{\partial^2 \psi(u_{m,n},\theta)}{\partial \theta^2} = \theta \frac{\partial \eta(u_{m,n},\theta)}{\partial \theta}. \tag{5} $$

“Instantaneous” means that these quantities change when the thermodynamic state changes itself. The traditional constant tensor of the linear elasticity is just the values of these tensors in reference (usually stress-free or hydrostatic) state.

The internal energy density function $e(u_{k,l},\eta)$ plays a particularly important role when dealing with adiabatic processes in which the entropy density remains unchanged in each material particle. Its first derivatives are given by the thermodynamic identities in equation 3.

The instantaneous adiabatic elastic modules ${}_{\eta}c^{ijkl}$ are defined by the second derivatives of the specific internal energy density: ${}_{\eta}c^{ijkl} \equiv \partial^2 e(u_{m,n},\eta)/\partial u_{i,j}\partial u_{k,l}$.

The first and second laws of thermodynamics imply certain thermodynamic identities binding the second derivatives of the two main thermodynamic potentials. In particular, these identities imply the following useful thermodynamic relationships:

$$ {}_{\eta}c^{ijkl} = {}_{\theta}c^{ijkl} + \frac{\theta}{c_u}\beta^{ij}\beta^{kl}\ , \tag{6} $$

and

$$ \frac{\partial^2 e(u_{k,l},\eta)}{\partial \eta^2} = -\frac{\theta}{c_u},\ \frac{\partial^2 e(u_{k,l},\eta)}{\partial \eta \partial u_{i,j}} = -\frac{\theta}{c_u}\beta^{ij}\ . \tag{7} $$

2.3 The Exact Nonlinear System of Equilibrium Equations for Isentropic Loading

We assume that the original stress-free state of the body has a uniformly distributed entropy $\eta = \eta_0$. We will also assume that in the loading processes, the entropy distribution remains unchanged. The following equilibrium conditions should then be satisfied within the bulk of the body and at its stress-free boundary, respectively:

$$\frac{\partial}{\partial x^j}\frac{\partial e(u_{m,n},\eta_0)}{\partial u_{i,j}} = 0\,, \tag{8}$$

and

$$\left.\frac{\partial e(u_{m,n},\eta_0)}{\partial u_{i,j}}\right|_S n_j = 0\,. \tag{9}$$

After solving equations 8 and 9, i.e., after determining the vector field of displacements $u_i(x)$,the equilibrium distributions of the stress tensor p^{ji} and the absolute temperature can be calculated with the help of the thermodynamic identities in equation 3:

$${}_{ad}p^{ij}\left(u_{k,l}\right) = \frac{\partial e(u_{k,l},\eta_0)}{\partial u_{i,j}}\ , \tag{10}$$

and

$${}_{ad}\theta(u_{k,l}) \equiv -\frac{\partial e(u_{k,l},\eta_0)}{\partial \eta}\,. \tag{11}$$

3. The Equilibrium System in Linear Approximation

Assume that the loading experiment can be adequately described in the framework of quadratic approximation of thermodynamic potentials near the stress-free configuration. In particular, this means that the free energy density can be approximated by the following relationship:

$$\begin{aligned}\psi(u_{m,n},\theta) &\approx \frac{1}{2}\frac{\partial^2\psi(0,\theta_0)}{\partial u_{i,j}\partial_{k,l}}u_{i,j}u_{k,l} + \frac{\partial^2\psi(0,\theta_0)}{\partial u_{i,j}\partial\theta}u_{i,j}T + \frac{1}{2}\frac{\partial^2\psi(u_{m,n},\theta_0)}{\partial\theta^2}T^2\\ &= \frac{1}{2}\,{}_{\theta}C^{ijkl}u_{i,j}u_{k,l} + B^{ij}u_{i,j}T - \frac{1}{2}\frac{C_u}{\theta_0}T^2,\end{aligned} \tag{12}$$

where tensors ${}_{\theta}C^{ijkl} \equiv {}_{\theta}c^{ijkl}(0,\theta_0)$, $B^{kl} \equiv \beta^{kl}(0,\theta_0)$, and $C_u \equiv c_u\left(0,\theta_0\right)$ play the roles of the second-order instantaneous tensors in the ground configuration; $T \equiv \theta - \theta_0$ is the notation of deviation of the absolute temperature.

Equation 12 leads to the following linear expansion of the stress tensor p^{ij}:

$$p^{ji}(u_{m,n},T) \approx {}_{\theta}C^{ijkl}u_{k,l} + B^{ij}T\,, \tag{13}$$

the bulk equilibrium equations,

$${}_{\theta}C^{ijkl}u_{k,jl} + B^{ij}T_{,j} = 0\,, \tag{14}$$

and boundary conditions at any traction-free interface S,

$$\left({}_{\theta}C^{ijkl}u_{k,l} + B^{ij}T\right)_S n_j = 0\,. \tag{15}$$

When dealing with adiabatic loading, it is more convenient to work with the following bulk and boundary equations of equilibrium:

$${}_{\eta}C^{ijkl}u_{k,jl} = 0\,, \tag{16}$$

and

$${}_{\eta}C^{ijkl}u_{k,l}\Big|_S n_j = 0\,. \tag{17}$$

The corresponding stress tensor and temperature change are given by the following formulas:

$${}_{ad}p^{ij} \approx {}_{\eta}C^{ijkl}u_{k,l}\,, \tag{18}$$

and

$${}_{ad}T = -\frac{\partial e(0,\eta_0)}{\partial \eta u_{k,l}}u_{k,l}\,. \tag{19}$$

With the help of the identities in equation 7, equation 19 can be rewritten as

$${}_{ad}T = \frac{\theta_0}{c_u}B^{kl}u_{k,l}\,. \tag{20}$$

Using relationships equations 6 and 20, equation 18 can be rewritten as follows:

$${}_{ad}p^{ij}\left(u_{k,l}\right) \approx {}_{\eta}C^{ijkl}u_{k,l} = \left({}_{\theta}C^{ijkl} + \frac{\theta_0}{C_u}B^{ij}B^{kl}\right)u_{k,l} = {}_{\theta}C^{ijkl}u_{k,l} + {}_{ad}TB^{ij}\,, \tag{21}$$

as it follows from equation 13.

Isotropic Elastic Substance

In the case of isotropic elastic substance, the tensors ${}_{\theta}C^{ijkl}$ and B^{ij}, by definition, have the following particularly simple structures:

$$ {}_{\theta}C^{ijkl} = \lambda_{\theta}\delta^{ij}\delta^{kl} + \mu_{\theta}\left(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}\right),\ B^{ij} = -K_{\theta}\alpha\delta^{ij}\,, \tag{22} $$

where λ_{θ} and μ_{θ} are the Lame isothermal elastic modules, $K_{\theta} \equiv \lambda_{\theta} + 2\mu_{\theta}/3$ is the isothermal module of volumetric compression, and α is the coefficient of thermal expansion. The definitions in equation 22 imply

$$ p^{ij} = \lambda_{\theta}\delta^{ij}u^{k}_{.,k}\delta^{kl} + 2\mu_{\theta}u^{i\ j}_{(.,.)} - K_{\theta}\alpha\delta^{ij}T\,. \tag{23} $$

Some formulas of thermodynamics of isotropic solids become more convenient when using the isothermal and isentropic Poisson ratios ν_{θ} and ν_{η}:

$$ \nu_{\theta} \equiv \frac{\lambda_{\theta}}{2\left(\lambda_{\theta} + \mu_{\theta}\right)},\ \nu_{\eta} \equiv \frac{\lambda_{\eta}}{2\left(\lambda_{\eta} + \mu_{\eta}\right)}\,. \tag{24} $$

Formulas in equation 24 imply the following useful identities:

$$ \lambda_{\theta} = \mu_{\theta}\frac{2\nu_{\theta}}{1-2\nu_{\theta}},\ \lambda_{\eta} = \mu_{\eta}\frac{2\nu_{\eta}}{1-2\nu_{\eta}}, K_{\theta} = \frac{2\mu}{3}\frac{1+\nu_{\theta}}{1-2\nu_{\theta}}, K_{\eta} = \frac{2\mu}{3}\frac{1+\nu_{\eta}}{1-2\nu_{\eta}}\,, $$

$$ K_{\theta} = \lambda_{\theta} + \frac{2}{3}\mu_{\theta} = \frac{2\mu_{\theta}}{3}\frac{1+\nu_{\theta}}{1-2\nu_{\theta}},\ K_{\eta} = \lambda_{\eta} + \frac{2}{3}\mu_{\eta} = \frac{2\mu_{\eta}}{3}\frac{1+\nu_{\eta}}{1-2\nu_{\eta}}\,. \tag{25} $$

Using equation 22, we can rewrite equations 20 and 21 as follows:

$$ {}_{ad}T = -\frac{\theta_0}{C_u}K_{\theta}\alpha u^{k\cdot}_{.k}\,, \tag{26} $$

and

$$ {}_{ad}p^{ij} \approx \lambda_{\theta}\delta^{ij}u^{k}_{.,k}\delta^{kl} + 2\mu_{\theta}u^{i\ j}_{(.,.)} - K_{\theta}\alpha\ {}_{ad}T\delta^{ij}\,. \tag{27} $$

With the help of equation 22, equations 6 and 7 can be rewritten as follows:

$$ \begin{aligned} {}_{\eta}C^{ijkl} &= {}_{\theta}C^{ijkl} + \frac{\theta_0}{C_u}K_{\theta}^2\alpha^2\delta^{ij}\delta^{kl} = \lambda_{\theta}\delta^{ij}\delta^{kl} + \mu_{\theta}\left(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}\right) + \frac{\theta_0}{C_u}K_{\theta}^2\alpha^2\delta^{ij}\delta^{kl} \\ &= \left(\lambda_{\theta} + \frac{\theta_0}{C_u}K_{\theta}^2\alpha^2\right)\delta^{ij}\delta^{kl} + \mu_{\theta}\left(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}\right), \end{aligned} \tag{28} $$

and

$$\frac{\partial^2 e(0,\eta_0)}{\partial \eta^2} = -\frac{\theta_0}{C_u}, \ \frac{\partial^2 e(0,\eta_0)}{\partial \eta \partial u_{i,j}} = \frac{\theta_0}{C_u} K_\theta \alpha \delta^{ij} \quad . \tag{29}$$

Introducing the isentropic Lame modules λ_η and μ_η according to the formulas

$$\lambda_\eta \equiv \lambda_\theta + \frac{\theta_0}{C_u} K_\theta^2 \alpha^2 \text{ and } \mu_\eta \equiv \mu_\theta , \tag{30}$$

we can rewrite equation 28 as follows:

$${}_\eta C^{ijkl} = \lambda_\eta \delta^{ij} \delta^{kl} + \mu_\eta \left(\delta^{ik} \delta^{jl} + \delta^{il} \delta^{jk} \right). \tag{31}$$

According to equation 27, the stresses in the adiabatic loading take the familiar form as follows:

$${}_{ad} p^{ij} \approx \lambda_\eta \delta^{ij} u^k_{.,k} \delta^{kl} + 2\mu_\eta u^{i\ j}_{(.,.)} . \tag{32}$$

Equation 30 implies

$$K_\eta \equiv K_\theta + \frac{\theta_0}{C_u} K_\theta^2 \alpha^2 , \ \frac{K_\eta}{K_\theta} \equiv \frac{C_u + K_\theta \theta_0 \alpha^2}{C_u} . \tag{33}$$

Also, we would like to indicate the thermodynamic identities which include the specific heat capacity C_p at fixed pressure:

$$C_p - C_u = K_\theta \theta_0 \alpha^2 , K_\eta \equiv K_\theta + \frac{\theta_0}{C_u} K_\theta^2 \alpha^2 , \ \frac{K_\eta}{K_\theta} \equiv \frac{C_p}{C_u} . \tag{34}$$

4. Temperature Distribution in Isotropic Elastic Space

4.1 Adiabatic Nonhydrostatic Loading of Unbounded Isotropic Space

This case is based on the analysis of equation 27, which can be rewritten in the following form:

$$\lambda_\theta \delta^{ij} u^k_{,k} + 2\mu_\theta u^{(i\ j)}_{.,.} = {}_{ad} p^{ij} + K_\theta \alpha_{ad} T \delta^{ij} . \tag{35}$$

Equation 35 implies the following:

$${}_{ad} u^k_{,k} = \frac{1}{3K_\theta} {}_{ad} p^{k.}_{.k} + \alpha_{ad} T = \frac{1}{3K_\eta} {}_{ad} p^{k.}_{.k} \ . \tag{36}$$

Combining equations 26 and 36, we get the following:

$$ {}_{ad}T = -\frac{\theta_0 \alpha}{C_u + \theta_0 \alpha^2 K_\theta} \frac{{}_{ad}\, p^{k\cdot}_{.k}}{3} = -\frac{\theta_0 \alpha}{C_p} \frac{{}_{ad}\, p^{k\cdot}_{.k}}{3}. \tag{37} $$

Using equations 34 and 37, we have more convenient formulas:

$$ {}_{ad}T = -\frac{\theta_0 \alpha}{C_u} \frac{K_\theta}{K_\eta} \frac{{}_{ad}\, p^{k\cdot}_{.k}}{3} = -\frac{\theta_0 \alpha}{C_u} \frac{1+\nu_\theta}{1-2\nu_\theta} \frac{1-2\nu_\eta}{1+\nu_\eta} \frac{{}_{ad}\, p^{k\cdot}_{.k}}{3}. \tag{38} $$

4.2 Formal Statement of the Boundary Value Problem

Summarizing the previously mentioned relationships, we arrive at the boundary value problem (BVP) in linear approximation.

The bulk equations of equilibrium:

$$ \left[\lambda_\eta \delta^{ij} \delta^{kl} + \mu_\eta \left(\delta^{ik} \delta^{jl} + \delta^{il} \delta^{jk} \right) \right] u_{k,jl} = 0. \tag{39} $$

The condition at the hole's boundary:

$$ \left[\lambda_\eta \delta^{ij} \delta^{kl} + \mu_\eta \left(\delta^{ik} \delta^{jl} + \delta^{il} \delta^{jk} \right) \right] u_{k,l} \Big|_S n_j = 0. \tag{40} $$

The conditions at infinity:

$$ u_i \to \kappa_{ij} x^j \ \ at \ \ |x^m| \to \infty. \tag{41} $$

After solving the BVP in equations 39–41, the impact-induced temperature change T can be found with the help of equation 26.

4.3 Formal Solution of the BVP in Equations 39–41

We will be looking for a solution of the system in equations 39–41 in the following solution:

$$ \begin{aligned} u_i &= \kappa_{ij} x^j - 4(1-\nu_\eta) D_{ip} \left(\frac{1}{r} \right)_{,\cdot\cdot}^{\ \ p} + D_{pq} \left(r + C \frac{1}{r} \right)_{,i\cdot\cdot}^{\ \ \cdot\, pq} = \kappa_{ij} x^j + 4(1-\nu_\eta) D_{ip} \frac{1}{r^3} x^p \\ &+ D_{pq} \left\{ \left(\delta^{pq} x_i + \delta_i^p x^q + \delta_i^q x^p \right) \left(C \frac{3}{r^5} - \frac{1}{r^3} \right) - x^p x^q x_i \left(C \frac{15}{r^7} - \frac{3}{r^5} \right) \right\}, \end{aligned} \tag{42} $$

where C and D_{pq} are the coordinate-independent scalar and symmetric tensor, respectively, and $r \equiv \sqrt{\delta_{ij} x^i x^j}$ and $\nu \equiv \lambda_\eta / 2\left(\lambda_\eta + \mu_\eta \right)$ are the adiabatic Poisson ratios.

A somewhat routine but cumbersome calculation leads to the following formulas:

$$u_i = \kappa_{ij} x^j + 4(1-\nu_\eta) D_{ip} \frac{1}{r^3} x^p$$
$$+ D_{pq} \left\{ \left(\delta^{pq} x_i + \delta_i^p x^q + \delta_i^q x^p \right) \left(C \frac{3}{r^5} - \frac{1}{r^3} \right) - \delta_{ir} x^p x^q x^r \left(C \frac{15}{r^7} - \frac{3}{r^5} \right) \right\}, \tag{43}$$

and

$$u_{i,j} = \kappa_{ij} - 4(1-\nu_\eta) D_{ip} \left(\frac{3}{r^5} \delta_{jr} x^p x^r - \frac{1}{r^3} \delta_j^p \right)$$
$$+ D_{pq} \left\{ \begin{array}{l} \left(\delta^{pq} \delta_{ij} + \delta_i^p \delta_j^q + \delta_i^q \delta_j^p \right) \left(C \frac{3}{r^5} - \frac{1}{r^3} \right) + 3\delta_{jr} x^r \left(\delta^{pq} \delta_{im} x^m + \delta_i^p x^q + \delta_i^q x^p \right) \left(\frac{1}{r^5} - C \frac{5}{r^7} \right) \\ -3\left(\delta_j^p x^q \delta_{ir} x^r + x^p \delta_j^q \delta_{ir} x^r + x^p x^q \delta_{ij} \right) \left(C \frac{5}{r^7} - \frac{1}{r^5} \right) - 15 x^p x^q \delta_{ir} x^r \delta_{jq} x^q \left(\frac{1}{r^7} - C \frac{7}{r^9} \right) \end{array} \right\}. \tag{44}$$

The formula of the stresses is even more cumbersome:

$$\frac{1}{\mu} p_{ij} = \frac{2\nu_\eta}{1-2\nu_\eta} \delta_{ij} \left[\kappa_{.k}^k + 2(1-2\nu_\eta) D_{.k}^k \frac{1}{r^3} \right] - 12\nu \delta_{ij} \delta_{km} D_{.l}^k x^l x^m \frac{1}{r^5}$$
$$+ 2\kappa_{(ij)} - 4(1-\nu_\eta) \left[D_{ik} \left(\frac{3}{r^5} x^k \delta_{jm} x^m - \frac{1}{r^3} \delta_j^k \right) - D_{jk} \frac{1}{r^3} \delta_i^k \right] - 4(1-\nu_\eta) D_{jk} \frac{3}{r^5} \delta_{im} x^k x^m$$
$$+ 2D_{pq} \left[\left(\delta^{pq} \delta_{ij} + 2\delta_i^{(p} \delta_j^{q)} \right) \left(C \frac{3}{r^5} - \frac{1}{r^3} \right) - 3\delta_{jm} x^m \left(\delta^{pq} \delta_{ik} x^k + 2\delta_i^{(p} x^{q)} \right) \left(C \frac{5}{r^7} - \frac{1}{r^5} \right) \right]$$
$$+ 2D_{pq} \left[15 \delta_{ik} \delta_{jm} x^p x^q x^m x^k \left(C \frac{7}{r^9} - \frac{1}{r^7} \right) - 3\left(2\delta_j^{(p} x^{q)} \delta_{im} x^m + x^p x^q \delta_{ij} \right) \left(C \frac{5}{r^7} - \frac{1}{r^5} \right) \right]. \tag{45}$$

Straightforward calculations, based on equation 43, show that the asymptotic condition in equation 41 at infinity is satisfied automatically. More cumbersome calculations show that the bulk equilibrium in equation 39 will be satisfied automatically as well. Lastly, straightforward calculations with equation 45 show that the boundary condition in equation 40 will be satisfied if the constant C assumes the value of

$$C = \frac{1}{5} R^2 \tag{46}$$

and the symmetric tensor satisfies the following linear algebraic system of equations:

$$\left(\frac{2-5\nu_\eta}{5} \delta^{pq} \delta_{ij} + \frac{7-5\nu_\eta}{5} 2\delta_i^{(p} \delta_j^{q)} \right) D_{pq} = \frac{R^3}{2\mu} p_{ij}^\infty, \tag{47}$$

where p_{ij}^{∞} is the value of the stress tensor at infinity

$$p_{ij}^{\infty} \equiv 2\mu\left(\frac{\nu_\eta}{1-2\nu_\eta}\kappa_{.k}^{k}\delta_{ij} + \kappa_{(ij)}\right). \tag{48}$$

Equation 48 implies the following useful relationship:

$$p_{.k}^{\infty k} = 2\mu\frac{1+\nu_\eta}{1-2\nu_\eta}\kappa_{.k}^{k}\,,\ \kappa_{.k}^{k} = \frac{1}{2\mu}\frac{1-2\nu_\eta}{1+\nu_\eta}p_{.k}^{\infty k}. \tag{49}$$

Equation 47 has the following solution:

$$D_{pq} = -\frac{R^3}{\mu}\frac{2-5\nu_\eta}{4\left(4-5\nu_\eta\right)\left(7-5\nu_\eta\right)}p_{k\,.}^{\infty k}\delta_{pq} + \frac{R^3}{\mu}\frac{5}{4\left(7-5\nu_\eta\right)}p_{pq}^{\infty}. \tag{50}$$

4.4 Temperature Distribution Around a Spherical Hole

Combining the formula of the displacement gradient in equation 44 with equations 46 and 49, we get

$$u_{.,k}^{k} = \kappa_{.k}^{k} + 2(1-2\nu_\eta)D_{qp}\frac{1}{r^3}\left(\delta^{qp} - \frac{3}{r^2}x^p x^q\right). \tag{51}$$

Combining equations 49 and 51, we arrive at the following formula of the local volume change:

$$u_{.,k}^{k} = \frac{1}{2\mu}\frac{1-2\nu_\eta}{1+\nu_\eta}p_{.k}^{\infty k} + (1-2\nu_\eta)\frac{2}{r^3}D_{pq}\left(\delta^{pq} - \frac{3}{r^2}x^p x^q\right). \tag{52}$$

Using equation 50, we can rewrite the formula of the divergence as follows:

$$u_{.,k}^{k} = \frac{1}{2\mu}\frac{1-2\nu_\eta}{1+\nu_\eta}p_{.k}^{\infty k} + \frac{R^3}{r^3}\frac{1}{\mu}\frac{5(1-2\nu_\eta)}{2\left(7-5\nu_\eta\right)}\left(\delta^{ij} - \frac{3}{r^2}x^i x^j\right)p_{ij}^{\infty}. \tag{53}$$

Finally, substituting equation 53 in equation 26 and using equation 25, we arrive at the following formula of the temperature change:

$${}_{ad}T = -\frac{\alpha\theta_0}{3C_u}\frac{1+\nu_\theta}{1-2\nu_\theta}\frac{1-2\nu_\eta}{1+\nu_\eta}\left[\delta^{ij} + 5\frac{R^3}{r^3}\frac{1+\nu_\eta}{7-5\nu_\eta}\left(\delta^{ij} - \frac{3}{r^2}x^i x^j\right)\right]p_{ij}^{\infty}. \tag{54}$$

With the help of identities in equations 25 and 34, equation 54 can also be rewritten as follows:

$$ {}_{ad}T = -\frac{\alpha\theta_0}{3C_u}\frac{K_\theta}{K_\eta}\left[\delta^{ij} + 5\frac{R^3}{r^3}\frac{1+\nu_\eta}{7-5\nu_\eta}\left(\delta^{ij} - \frac{3}{r^2}x^i x^j\right)\right]p_{ij}^{\infty} \tag{55} $$

$$ = -\frac{\alpha\theta_0}{3C_p}\left[\delta^{ij} + 5\frac{R^3}{r^3}\frac{1+\nu_\eta}{7-5\nu_\eta}\left(\delta^{ij} - \frac{3}{r^2}x^i x^j\right)\right]p_{ij}^{\infty} \ . $$

This is the main result of this report, which is discussed in the next section.

5. Discussion and Conclusion

Equations 54 and 55 are quite instructive and transparent. The first terms give the spatially uniform adiabatic temperature increase $T_{uniform}$ that would exist in the absence of the hole in the infinite elastic media:

$$ T_{uniform} = -\frac{\alpha\theta_0}{3C_p}p_{.\,i}^{\infty\, i} = -\frac{\alpha\theta_0}{3C_u}\frac{K_\theta}{K_\eta}p_{.\,i}^{\infty\, i} \ . \tag{56} $$

The second term gives a spatially nonuniform additional temperature increase T_{local} due to the presence of the hole:

$$ {}_{ad}T_{local}\left(x^i\right) = -\frac{5}{3}\frac{\alpha\theta_0}{C_p}\frac{1+\nu_\eta}{7-5\nu_\eta}\frac{R^3}{r^3}\left(\delta^{ij} - \frac{3}{r^2}x^i x^j\right)p_{ij}^{\infty} \ . \tag{57} $$

The temperature field T_{local} is localized in a small vicinity of the hole and decays as the inverse cube of r distance from the center of the hole. However, at the hole's boundary at $r = R$, both temperature fields $T_{uniform}$ and T_{local} are the same order of magnitude.

Remarkably, the local temperature field T_{local} identically vanishes when the solid undergoes the pressure-like external loading at infinity, i.e., when $p_{ij}^{\infty} \equiv -p_{ext}\delta_{ij}$. On the other hand, the uniform temperature field $T_{uniform}$ vanishes at shear-like loadings, i.e., when $p_{.i}^{\infty\, i} \equiv 0$. In this case, the local field T_{local} not only dominates but becomes the only existing temperature field induced by the adiabatic impact. In this case, it can be rewritten in the simpler form:

$$ {}_{ad}T_{local}\left(x^i\right) = \frac{\alpha\theta_0}{C_p}\frac{5\left(1+\nu_\eta\right)}{7-5\nu_\eta}\frac{R^3}{r^3}r^i r^j p_{ij}^{\infty} \ , \tag{58} $$

where $r^i(x^k) \equiv x^i / r$ are the Cartesian components of a unit vector having the same direction as the radius-vector $\vec{r}(x^k)$ drawn from the center of the hole to the point x^i.

Equations 57 and 58 become even more transparent in terms of the three principal stresses, p^L of the tensor p_{ij}^{∞}, corresponding to the three principal directions with the unit vectors l_i^L. In terms of the principal quantities, the tensor p_{ij}^{∞} can be presented in the following form:

$$p_{ij}^{\infty} = \sum_{L=1}^{3} p^L l_i^L l_j^L . \tag{59}$$

Introducing the directional cosines $\cos\theta_L \equiv r^i l_i^L$, we can rewrite equation 57 as follows:

$$T_{local}(x) = \frac{\alpha\theta_0}{C_p} \frac{5\left(1+\nu_\eta\right)}{7-5\nu_\eta} \frac{R^3}{r^3} \sum_{L=1}^{3} p_L \sin^2\theta_L . \tag{60}$$

In particular, for the uniaxial stressing along the axis x^1, equations 56–60 give the following formula of the spatial distribution of the impact-induced temperature field:

$$_{ad}T = -5\frac{\alpha\theta_0}{C_p} \frac{1+\nu_\eta}{7-5\nu_\eta} p^1 \left(1 - \frac{R^3}{r^3} \sin^2\theta_1\right). \tag{61}$$

On the boundary of the hole, equation 61 gives the following formula:

$$_{ad}T\Big|_S = -\frac{\alpha\theta_0}{C_p} \frac{5\left(1+\nu_\eta\right)}{7-5\nu_\eta} p^1 \cos^2\theta_1 . \tag{62}$$

All these remarkable qualitative facts should be taken into account when discussing generation of hot spots in solids under the action of adiabatic impacts. The explicit solution can also be used for verification of numerical codes and planning experiments. At this stage, it is important to merge theoretical modeling with computer-based modeling and physical experiment.

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